Stochastic interacting particle systems: from applications to theory

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Abstract

One of the objectives of this lecture is to build a bridge between probability theory and applications. We will start from various questions, mainly related to our environment or industrial questions, model them using random variables, and then use probability theory to study them.

This lecture does not aim to be exhaustive, as probabilistic models and applications are highly diverse, and there are numerous probabilistic methods for studying them. We will focus on certain models of interacting particles and on the concept of propagation of chaos. We will review some key concepts related to the convergence of processes in order to study two types of stochastic particle systems in detail.

- Lecture 1: Models of interacting particle systems in various fields.

  Exercise 1, page 10
- Lecture 2: Kac's propagation of chaos.
- Lecture 3: Convergence of stochastic processes: tightness, Aldous' criterion, and so on.

  Exercise 2, page 27
- Lecture 4: Analysis of a diffusive interacting particle system.

  Exercise 3, page 29
Exercise 4, page 34
- Lecture 5: Analysis an interacting particle system with jumps.

  Exercise 5, page 38

For students who wish to receive credit for this course, the grade is divided into two parts:

- Attendance at class sessions: 10 points per lecture (please remember to sign the attendance sheet at each class);
- Submission of written assignments (at least 4 out of the 5 exercises): 50 points.

Assignments must be submitted by email to guerin.helene@uqam.ca (please make sure to include your **student ID number** both in your signature and in the names of your PDF files). The deadline for submitting assignments is **Tuesday, June 23, at 4:00 p.m.**

Please send your solutions, in English, as PDF files, either generated from a LaTeX source file or compiled from photos of your handwritten (legible) solutions. Only one PDF file per assignment is allowed (so if you use photos, please combine them into a single PDF file).

The grade will primarily reflect the quality of your writing and your ideas. Therefore, do not worry if you are unable to completely solve an exercise.

If you have any questions, please feel free to email me.

Contents

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A FEW NOTATIONS:

- $X \sim \mu$ means that the random variable follows the probability distribution μ: $\text{Law}(X) = \mu$;
- Δ and ∇ represent, respectively, the Laplacian operator and the gradient;
- ∂_t and ∂_x denote the partial derivatives with respect to time t and space x, respectively;
- the indicator function $\mathbb{1}_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$;
- $\mathcal{P}(E)$ denotes the set of probability measures on a Polish space E;
- $\mathcal{C}_b(E)$ denotes the space of bounded and continuous functions on E;
- $\langle \mu, \varphi \rangle = \int_E \varphi \mathrm{d}\mu$ for $\mu \in \mathcal{P}(E)$ and φ a test function on E;
- $\|\varphi\|_\infty = \sup_{x \in E} |\varphi(x)|$ for a function defined on E;
- $|x|$ denotes the Euclidean norm of $x \in \mathbb{R}^d$;
- $\mathcal{C}(\mathbb{R}_+, \mathbb{R}^d)$ denotes the space of continuous function defined on $\mathbb{R}_+$ with values in $\mathbb{R}^d$;
- $\mathbb{D}(\mathbb{R}_+, \mathbb{R}^d)$ denotes the Skorokhod space, i.e. the space of *càdlàg* functions (right-continuous functions with left limits), defined on $\mathbb{R}_+$ with values in $\mathbb{R}^d$;
- $z(t-) = \lim_{\substack{s \to t \\ s < t}} z(s)$ is the left limit of a function $z \in \mathbb{D}(\mathbb{R}_+, \mathbb{R}^d)$ in $t \in \mathbb{R}_+$,

References

[Ald78] David Aldous. Stopping times and tightness. *Ann. Probability*, 6(2):335–340, 1978.

[BGL01] Sergey G. Bobkov, Ivan Gentil, and Michel Ledoux. Hypercontractivity of Hamilton-Jacobi equations. *J. Math. Pures Appl. (9)*, 80(7):669–696, 2001.

[Bil99] Patrick Billingsley. *Convergence of probability measures*. Wiley Series in Probability and Statistics: Probability and Statistics. John Wiley & Sons, Inc., New York, second edition, 1999. A Wiley-Interscience Publication.

[BP87] Richard F. Bass and Étienne Pardoux. Uniqueness for diffusions with piecewise constant coefficients. *Probab. Theory Related Fields*, 76(4):557–572, 1987.

[BRTV98] S. Benachour, B. Roynette, D. Talay, and P. Vallois. Nonlinear self-stabilizing processes. I. Existence, invariant probability, propagation of chaos. *Stochastic Process. Appl.*, 75(2):173–201, 1998.

[CD22a] Louis-Pierre Chaintron and Antoine Diez. Propagation of chaos: A review of models, methods and applications. I. models and methods. *Kinetic and Related Models*, 15(6):895, 2022.

[CD22b] Louis-Pierre Chaintron and Antoine Diez. Propagation of chaos: A review of models, methods and applications. II. applications. *Kinetic and Related Models*, 15(6):1017, 2022.

[CMV03] José A. Carrillo, Robert J. McCann, and Cédric Villani. Kinetic equilibration rates for granular media and related equations: entropy dissipation and mass transportation estimates. *Rev. Mat. Iberoamericana*, 19(3):971–1018, 2003.

[DMGLP15] Anna De Masi, Antonio Galves, Eva Löcherbach, and E. Presutti. Hydrodynamic limit for interacting neurons. *J. Stat. Phys.*, 158(4):866–902, 2015.

[EK86] Stewart N. Ethier and Thomas G. Kurtz. *Markov processes*. Wiley Series in Probability and Mathematical Statistics: Probability and Mathematical Statistics. John Wiley & Sons, Inc., New York, 1986. Characterization and convergence.

[Fer02] E. Robert Fernholz. *Stochastic portfolio theory*, volume 48 of *Applications of Mathematics (New York)*. Springer-Verlag, New York, 2002. Stochastic Modelling and Applied Probability.

[FIKP13] E. Robert Fernholz, Tomoyuki Ichiba, Ioannis Karatzas, and Vilmos Prokaj. Planar diffusions with rank-based characteristics and perturbed Tanaka equations. *Probab. Theory Related Fields*, 156(1-2):343–374, 2013.

[FL16] Nicolas Fournier and Eva Löcherbach. On a toy model of interacting neurons. *Ann. Inst. Henri Poincaré Probab. Stat.*, 52(4):1844–1876, 2016.

[FPPZN25] Raphaël Forien, Guodong Pang, Etienne Pardoux, and Arsene Brice Zotsa-Ngoufack. Stochastic epidemic models with varying infectivity and waning immunity. *The Annals of Applied Probability*, 35(3):2175–2216, 2025.

[GZN25] Hélène Guérin and Arsene Brice Zotsa-Ngoufack. A stochastic epidemic model with memory of the last infection and waning immunity, 2025. arXiv:2505.00601.

[IKS13] Tomoyuki Ichiba, Ioannis Karatzas, and Mykhaylo Shkolnikov. Strong solutions of stochastic equations with rank-based coefficients. *Probab. Theory Related Fields*, 156(1-2):229–248, 2013.

[JR13] Benjamin Jourdain and Julien Reygner. Propagation of chaos for rank-based interacting diffusions and long time behaviour of a scalar quasilinear parabolic equation. *Stoch. Partial Differ. Equ. Anal. Comput.*, 1(3):455–506, 2013.

[JS03] Jean Jacod and Albert N. Shiryaev. *Limit theorems for stochastic processes*, volume 288 of *Grundlehren der mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, second edition, 2003.

[KS18] Praveen Kolli and Mykhaylo Shkolnikov. SPDE limit of the global fluctuations in rank-based models. *Ann. Probab.*, 46(2):1042 – 1069, 2018.

[Mal03] Florent Malrieu. Convergence to equilibrium for granular media equations and their Euler schemes. *Ann. Appl. Probab.*, 13(2):540–560, 2003.

[OV00] F. Otto and C. Villani. Generalization of an inequality by Talagrand and links with the logarithmic Sobolev inequality. *J. Funct. Anal.*, 173(2):361–400, 2000.

[SV06] Daniel W. Stroock and S. R. Srinivasa Varadhan. *Multidimensional diffusion processes*. Classics in Mathematics. Springer-Verlag, Berlin, 2006. Reprint of the 1997 edition.

[Szn91] Alain-Sol Sznitman. Topics in propagation of chaos. In *École d'Été de Probabilités de Saint-Flour XIX—1989*, volume 1464 of *Lecture Notes in Math.*, pages 165–251. Springer, Berlin, 1991.